

Math 116 Section 04

Midterm 4

Name _____

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Student Number _____

All solutions are to be presented on the paper in the space provided. The exam is closed book, no calculators. Time for the exam is 50 minutes.

(1) (**5 marks**) Evaluate $\int_3^\infty e^{-x} dx$

$$\begin{aligned} &= \lim_{t \rightarrow \infty} \int_3^t e^{-x} dx \\ &= \lim_{t \rightarrow \infty} -e^{-x} \Big|_3^t \\ &= -\lim_{t \rightarrow \infty} (e^{-t} - e^{-3}) \\ &= -(0 - e^{-3}) \\ &= e^{-3} \end{aligned}$$

(2) (**5 marks**) Prove $\lim_{x \rightarrow 2} (x + 1) = 3$.
Let $\epsilon > 0$ and find δ :

$$|(x + 1) - 3| < \epsilon$$

$$|x - 2| < \epsilon$$

So, let $\delta = \epsilon$. Then, the proof is:

$$|x - 2| < \delta = \epsilon$$

$$|(x + 1) - 3| < \epsilon$$

(3) (**5 marks**) Evaluate $\lim_{x \rightarrow 0} x^x$.

Let $y = x^x$. Then $\ln y = x \ln x$.

$$\begin{aligned} \lim_{x \rightarrow 0} x \ln x &= 0 \cdot -\infty && \lim_{x \rightarrow 0} \frac{\ln x}{\frac{1}{x}} \\ &= \frac{-\infty}{\infty} && \lim_{x \rightarrow 0} \frac{\frac{1}{x}}{\frac{-1}{x^2}} \\ &= && \lim_{x \rightarrow 0} -x \\ &= && 0 \end{aligned}$$

So, $\lim_{x \rightarrow 0} y = e^0 = 1$.

(4) (**5 marks**) Solve the initial value problem $\frac{dy}{dx} = x$, $x(0) = 1$

$$\frac{dy}{dx} = x$$

$$dy = xdx$$

$$\int dy = \int xdx$$

$$y = \frac{x^2}{2} + C$$

Using $x(0) = 1$, we get $1 = 0 + C$, so that the solution is $y = \frac{x^2}{2} + 1$.